Chaotic Magnetic Pendulum

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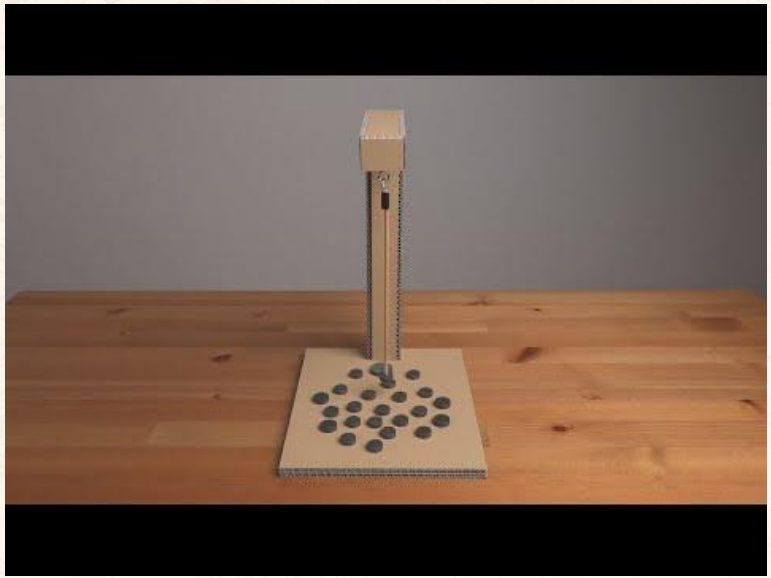
Problem

Consider a pendulum consisting of a magnetic bob attached to a string. If the pendulum is allowed to swing over a structure of permanent magnets, it will display complex motion.

Study the pendulum dynamics and its dependence on the number of permanent magnets and their arrangements

— International Physicist Tournament 2021



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Simulation Set Up

01

1. Assumptions in 2D simulation
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1. Assumptions in 2D Simulation

Small Angle
- Linear Restore
- Static Z

Monopole Magnets
- Like Charges

2. Set up magnets and forces

Force Model
- Componentwise
- Arrays

Static Base Forces
- Drag
- Gravity

```
def aY(x,y,vy):

  #Contribution from gravity
    ygrav = -K*y

  #Contribution from drag
    ydrag = -B*vy


  #Contribution from magnet one
    if y-MY[0] == 0:
        ymag1=0
    else:
        ymag1 = sign(MY[0]-y)*(J*S[0]*(abs(np.cos(np.arctan((x-MX[0])/(y-MY[0])))))/((x-MX[0])**2+(y-MY[0])**2+(Z)**2)**1.5
  #Contribution from magnet two
    if y-MY[1] == 0:
        ymag2=0
    else:
        ymag2 = sign(MY[1]-y)*(J*S[1]*(abs(np.cos(np.arctan((x-MX[1])/(y-MY[1])))))/((x-MX[1])**2+(y-MY[1])**2+(Z)**2)**1.5
  #Contribution from magnet three
    if y-MY[2] == 0:
        ymag3=0
    else:
        ymag3 = sign(MY[2]-y)*(J*S[2]*(abs(np.cos(np.arctan((x-MX[2])/(y-MY[2])))))/((x-MX[2])**2+(y-MY[2])**2+(Z)**2)**1.5

    return ygrav + ydrag + ymag1 + ymag2 + ymag3
```

3. Path Generation - Initial Conditions

- Set up empty arrays
  - N = Number of steps
- Insert initial conditions
  - Index 0

```
#Declare main arrays
t = np.empty(N) #Time array
x = np.empty(N) #x position array
y = np.empty(N) #y position array
vx = np.empty(N) #x velocity array
vy = np.empty(N) #y velocity array
```

```
#Initial conditions (start with no velocity)
t[0] = 0 #Initial time (in sec)
x[0] = 5 #Initial x position (in m)
y[0] = 1 #Initial y position (in m)
vx[0] = 0 #Initial x velocity (in m/s)
vy[0] = 0 #Initial y velocity (in m/s)
```

3. Path Generation - Loop

- Loop through N steps
  - i = index for each array
  - Update x, y values for every i
  - Use the equations of kinematics

```
for i in range(N-1):
        t[i+1] = t[i] + DT

        x[i+1] = x[i]+vx[i]*DT+(aX(x[i],y[i],vx[i])*(DT**2)*0.5)
        y[i+1] = x[i]+vy[i]*DT+(aY(x[i],y[i],vy[i])*(DT**2)*0.5)

        vx[i+1] = vx[i] + aX(x[i],y[i],vx[i])*DT
        vy[i+1] = vy[i] + aY(x[i],y[i],vy[i])*DT
```

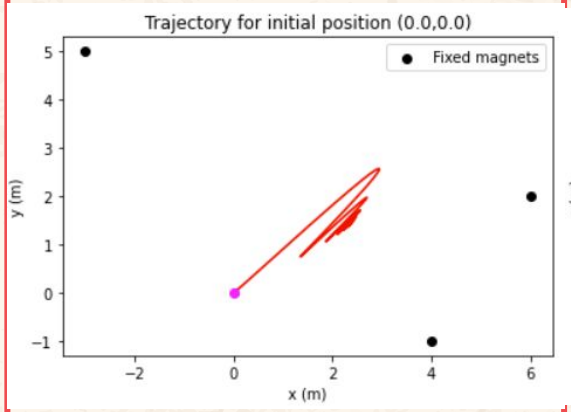
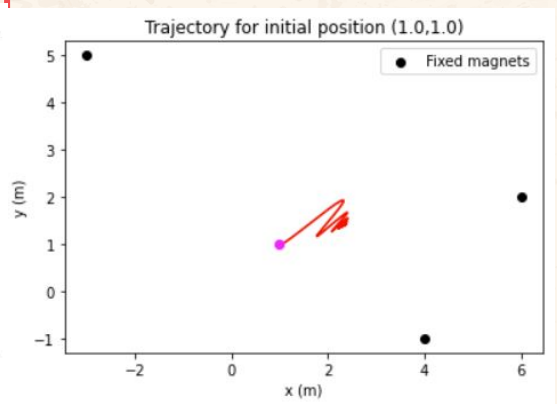
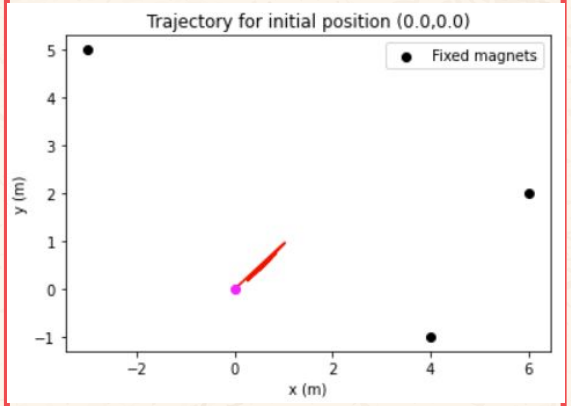
```
#Plot the trajectory
plt.plot(x,y,zorder=1,color='red')
plt.scatter(MX,MY,color='black',zorder=2,label='Fixed magnets')
plt.scatter(x[0],y[0],color='magenta',zorder=3)
title = 'Trajectory for initial position ('+str(x[0])+','+str(y[0])+')'
plt.title(title)
plt.xlabel('x (m)')
plt.ylabel('y (m)')
plt.legend()
plt.show()
```

Varying Conditions

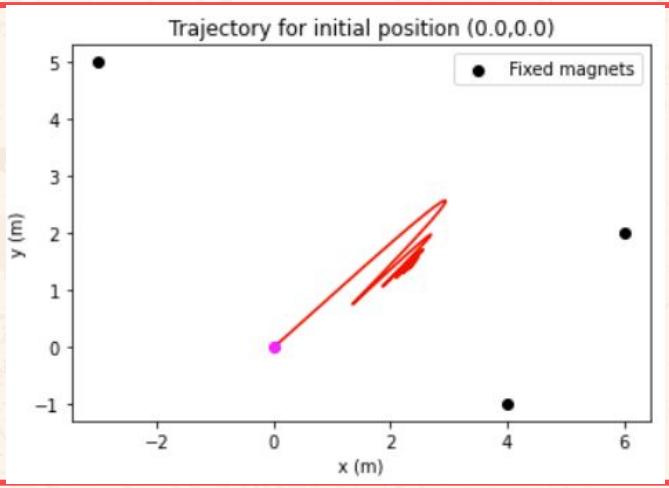
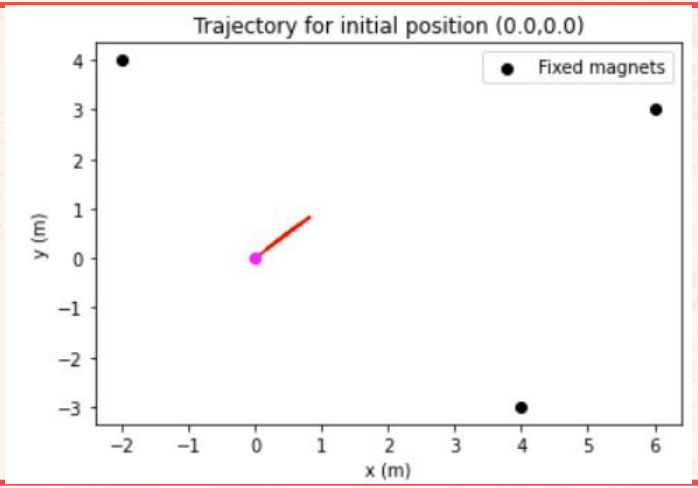
02

1. Initial Position
2. Positions of magnets
3. Strengths of magnets

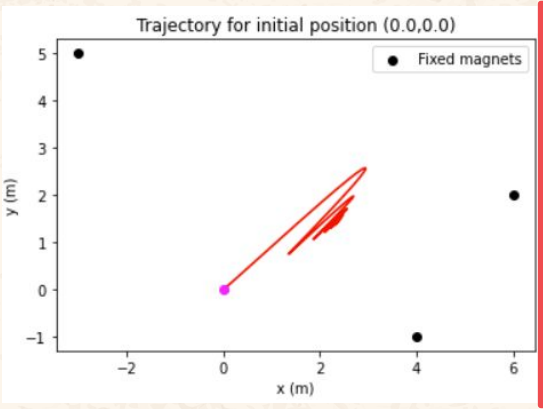
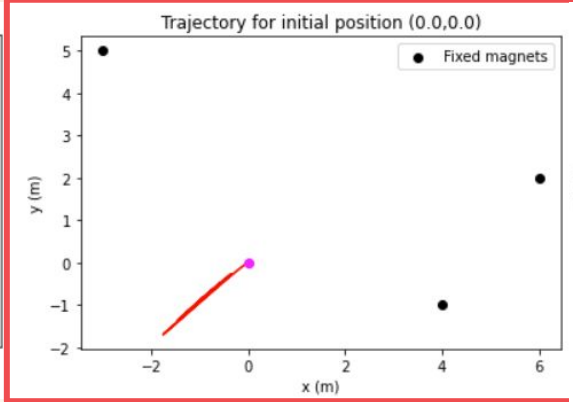
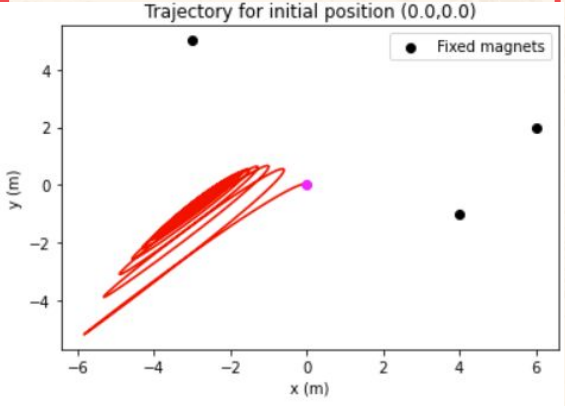
Initial Position

(0, 0, .25)	(1, 1, .25)	(0, 0, 5)
		

Positions of magnets

(-3,5/4,-1/6,2)	(4,-3/6,3/-2,4)
	

Strengths of magnets

(10, 10,10)	(-10, -10, -10)	(100, -40, -40)
		

Animation

- Loop through i to get a set of frames

```
# Initilize writer
metadata = dict(title='2D animation', artist='Matplotlib')
writer = FFMpegWriter(fps=50, metadata=metadata, bitrate=200000)
fig = plt.figure(dpi=200)

fig, ax = plt.subplots()

## SAVE AS MP4 ##
with writer.saving(fig, "magnet.mp4", dpi=200):
    for i in range(N-1):
        ax.clear()

        ax.plot(x,y,zorder=1,color='magenta')
        ax.scatter(MX,MY,color='black',zorder=2,label='Fixed magnets')
        ax.scatter(x[i],y[i],color='magenta',zorder=5)

        plt.draw()
        plt.pause(0.00001)
        writer.grab_frame()
```

Line of path

Permanent magnets

Pendulum bob

03

Animation

An interactive illustration of how the pendulum swings

Animation

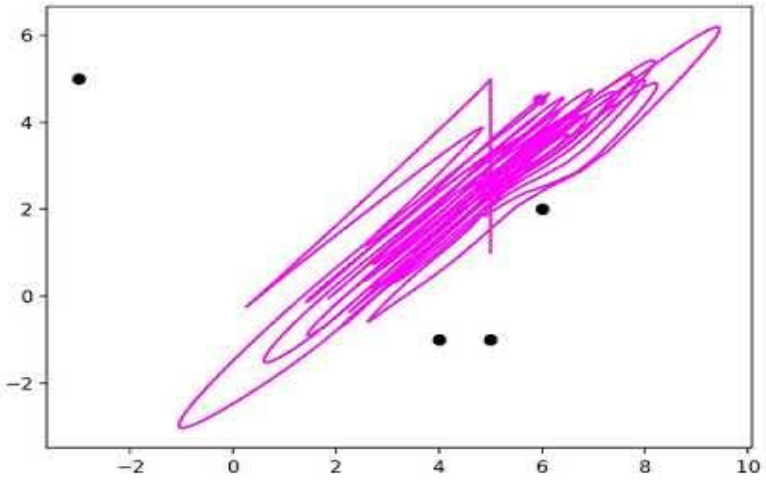
- Loop through i to get a set of frames

```
# Initilize writer
metadata = dict(title='2D animation', artist='Matplotlib')
writer = FFMpegWriter(fps=50, metadata=metadata, bitrate=200000)
fig = plt.figure(dpi=200)

fig, ax = plt.subplots()

## SAVE AS MP4 ##
with writer.saving(fig, "magnet.mp4", dpi=200):
    for i in range(N-1):
        ax.clear()

        ax.plot(x,y,zorder=1,color='magenta')
        ax.scatter(MX,MY,color='black',zorder=2,label='Fixed magnets')
        ax.scatter(x[i],y[i],color='magenta',zorder=5)

        plt.draw()
        plt.pause(0.00001)
        writer.grab_frame()
```

Line of path

Permanent magnets

Pendulum bob



Demonstration

Path above 4 magnets

Pendulum: (5, 1, 0)

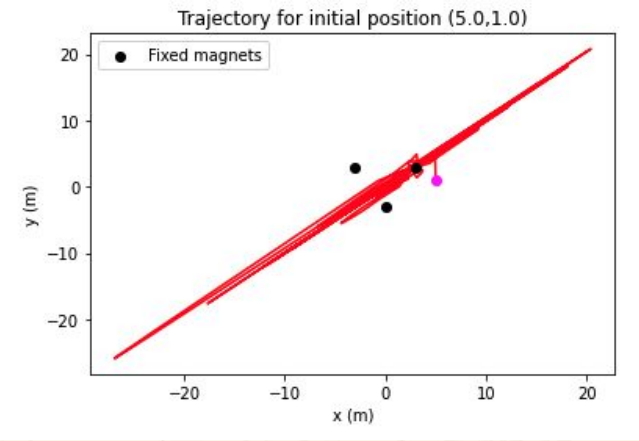
Magnets:(-3, 5, 0) (4, -1, 0) (6, 2, 0) (5, -1 , 0)

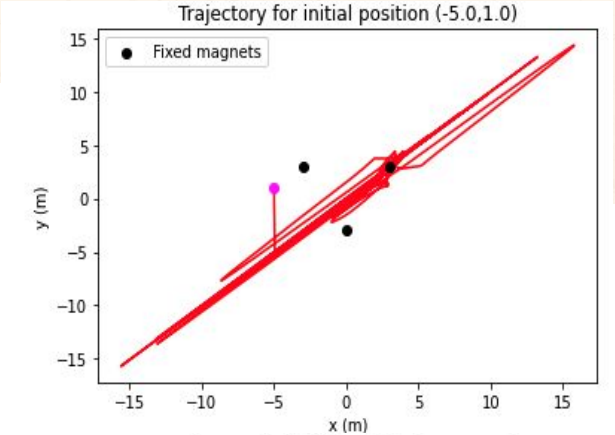
03

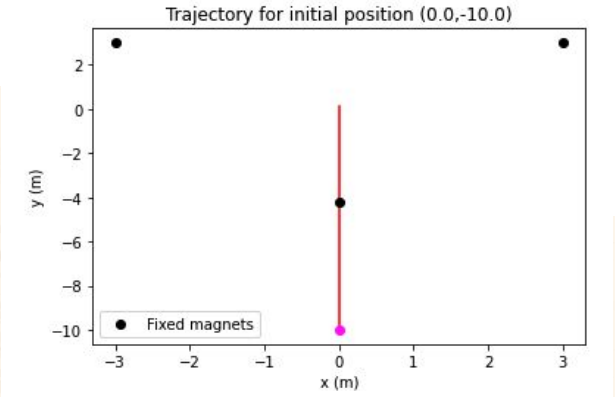
Basins of attraction

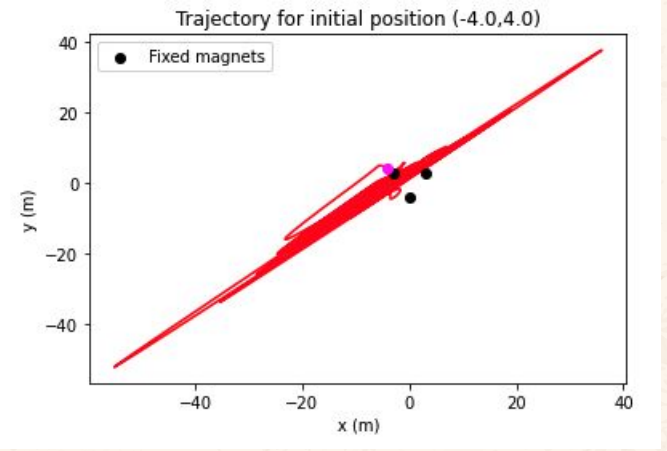
Regular patterns in chaotic behaviour

Patterns observed with special magnet positions

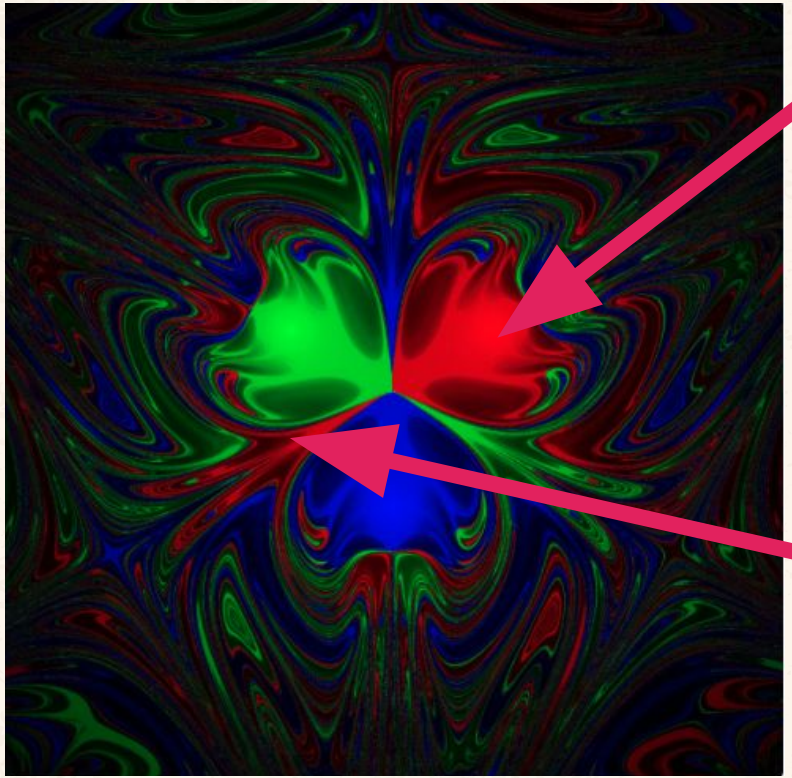


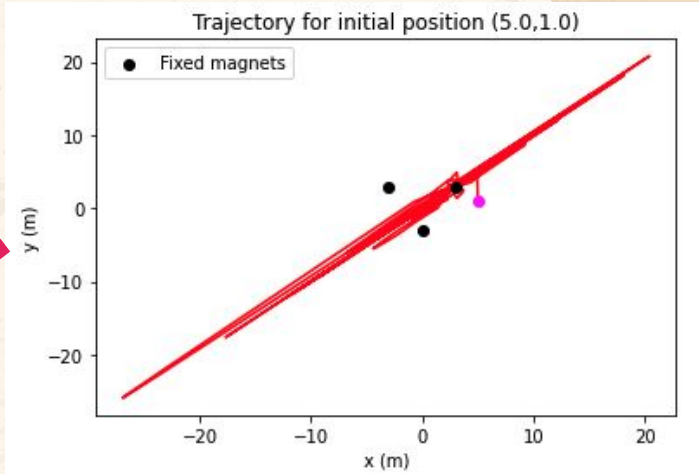


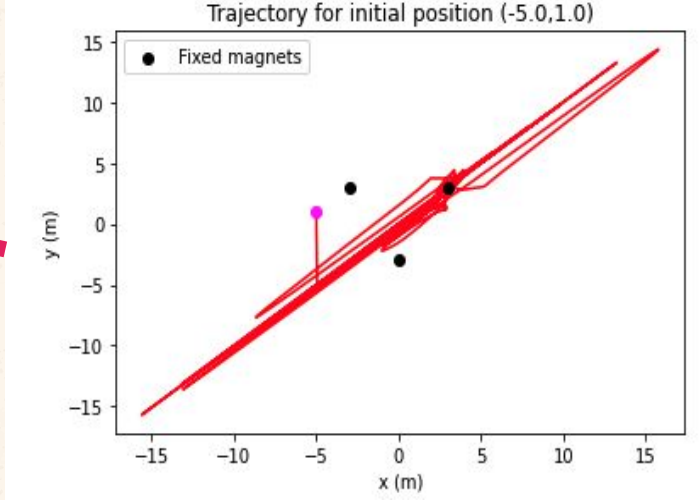




Basins of attraction







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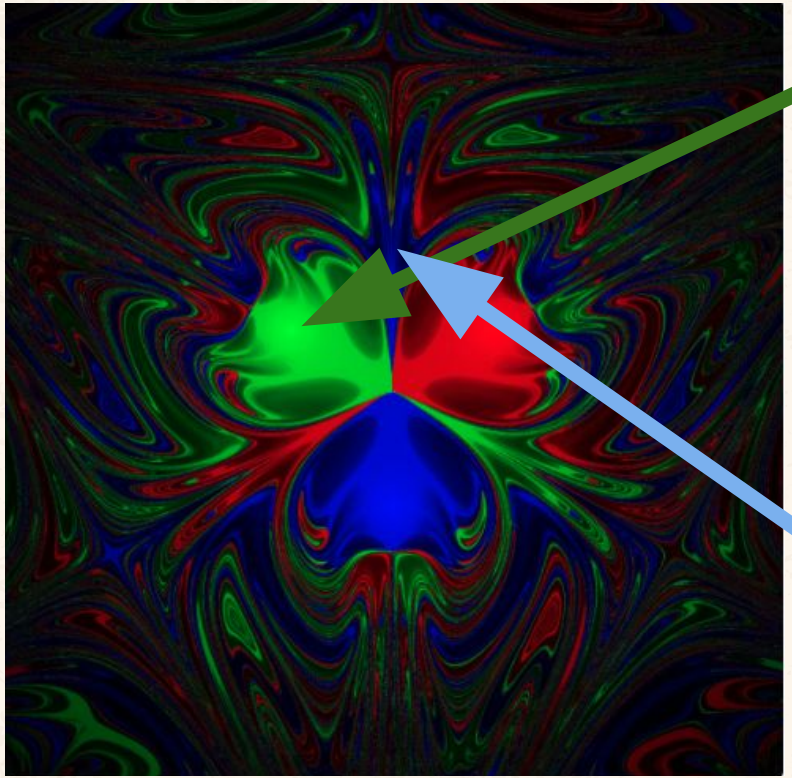

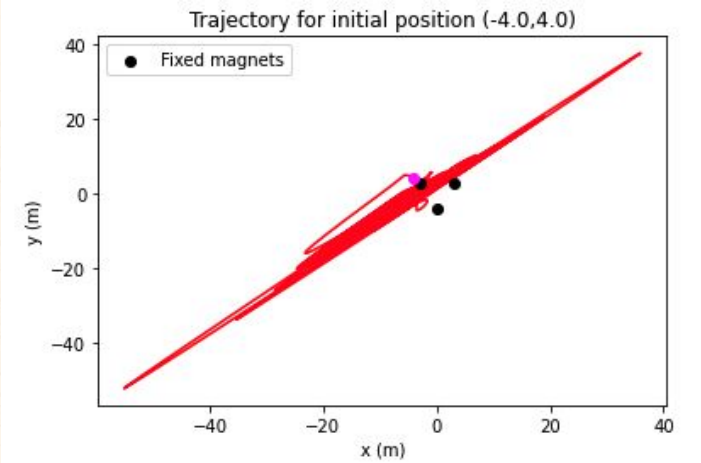

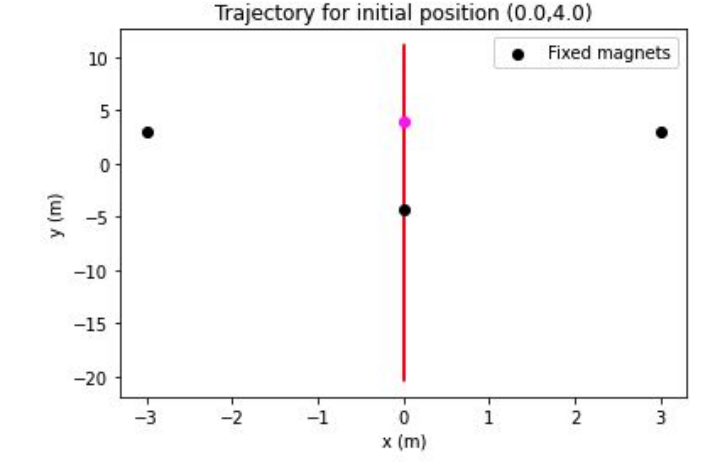

THANK YOU!

References:

https://iptnet.info/

https://compute.dawsoncollege.qc.ca/winter-2020/modelling-physical-systems-odes/magnetic-pendulum-trajectory/

https://www.math.hmc.edu/~dyong/math164/2006/win/finalreport.pdf

https://softologyblog.wordpress.com/2012/04/11/chaotic-oscillating-magnetic-pendulum-simulation/