Simulating the Rocket Equation

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Background Information

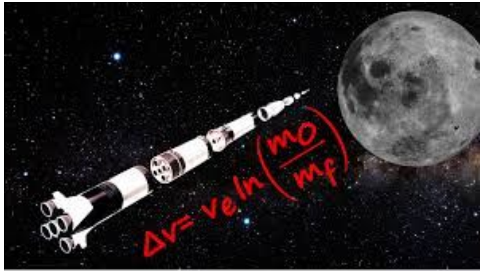
$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

where:

Δv	Change in velocity
v_e	Exhaust velocity
m_0	Initial (launch) mass
m_f	Final (dry) mass

- Imagine a motionless mass in deep space (no external forces acting on it)
  - This mass can move by expelling its own mass
  - The expelled mass has momentum in one direction, the remaining mass will gain momentum in the opposite direction
    - THE TOTAL MOMENTUM IS CONSTANT (conservation of momentum)
- The equation to represent this motion is called the rocket equation
  - $\Delta v = v_{exhaust} \ln(m_{initial}/m_{final})$
- In this experiment, we will simulate the rocket equation as well as analyze the impact of friction and gravity on the mass in question

$$\Delta v = v_e \ln \frac{m_o}{m_f}$$

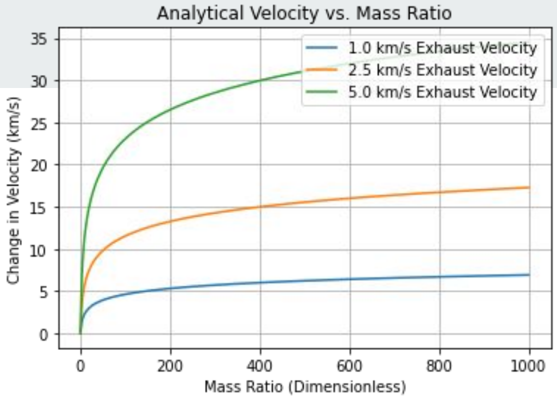


Methods and Techniques

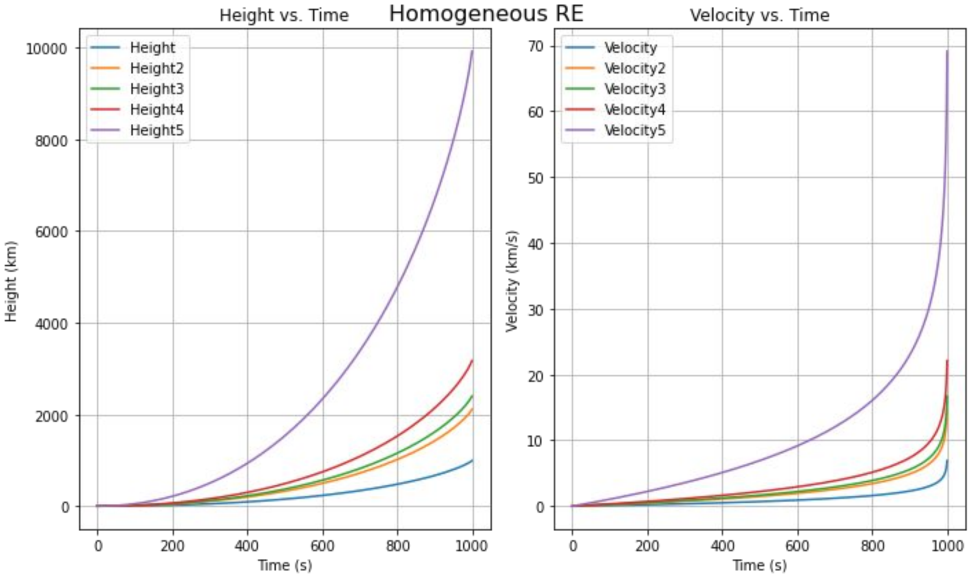
```
def full_RE(Y, t):
  y = Y[0]
  v = Y[1]
  dydt = v
  dvdt = -((v_exh * dmdt(t)) + (m(t)*g) + (b*(dydt)**2))/m(t)
  return [dydt,dvdt]
```

```
Y0 = [0,0]
Fsol = integrate.odeint(full_RE, Y0, t)
```

Findings and Results



Exhaust Velocities Impact on Velocity



It is apparent that a greater exhaust velocity will lead to a greater change in velocity for the mass overall.

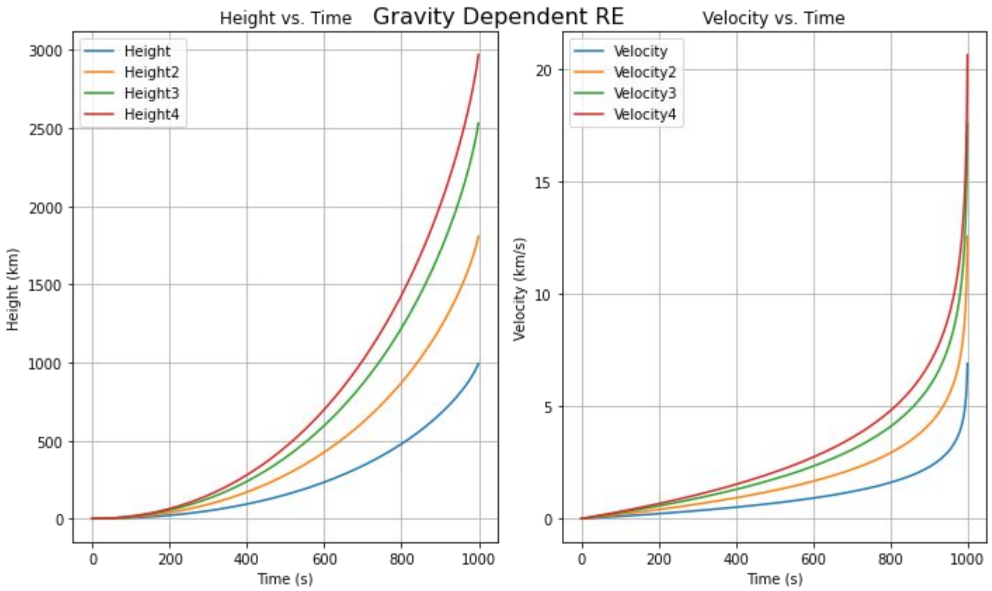
As the mass ratio increases, the change in velocity becomes less and less drastic

The increase in height of the rocket also exhibits a similar behavior.

This situation has no external forces acting on the rocket, the only term distinguishing its height and velocity is the changing exhaust velocity

Moving forward, we will plot what happens to the RE when we apply different external forces

Gravity Dependent Rocket Equation

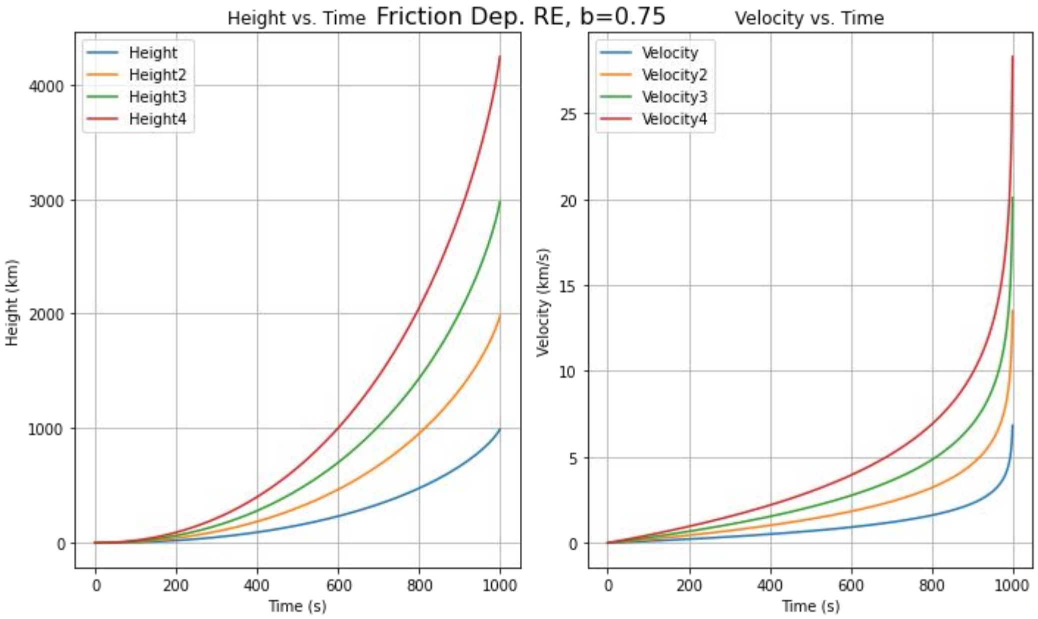


The total height and velocity of the rocket are much lower than that of the homogenous situation

(Note that we did not plot v_exh5 in this situation, it is an extreme case for the force-independent model)

The slopes of the graphs are similar to the homogenous RE, as gravity doesn't have too big of an impact on the rocket's height and velocity.

Friction's Impact on the RE (small constant)



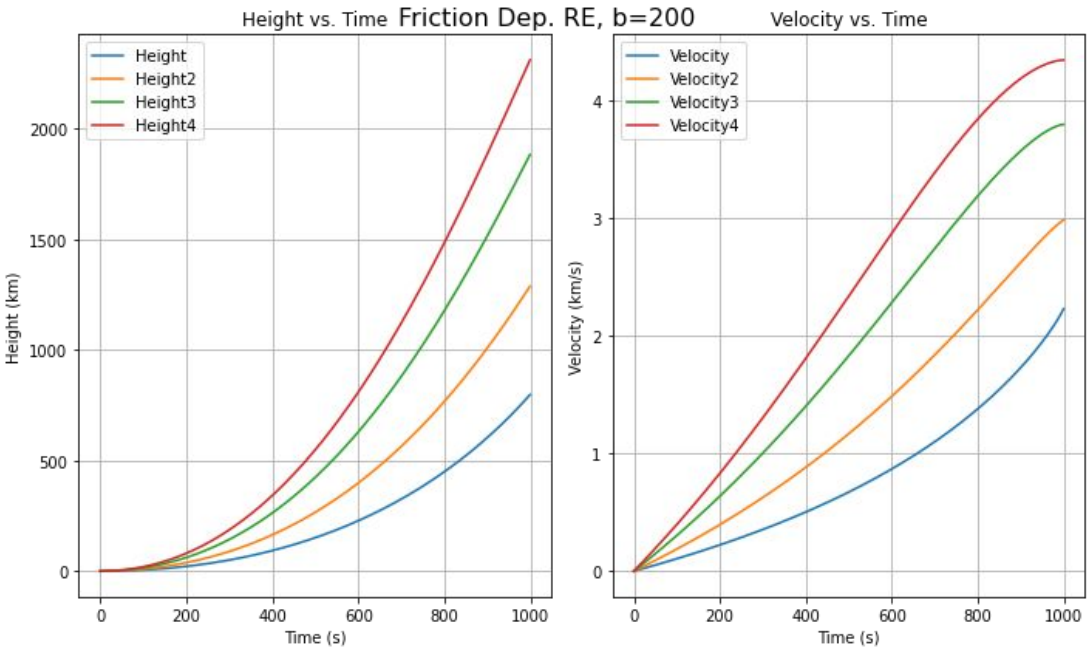
In this case, the friction constant (b) is 0.75

While this is a low number, it still has a large effect on the height and velocity of the rocket

In the homogenous case, the rocket was at ~3,500 km and ~25 km/s at 1000 seconds

In this case the rocket is at ~4000 km and ~30 km/s at 1000 seconds

Friction’s Impact on the RE (large constant)



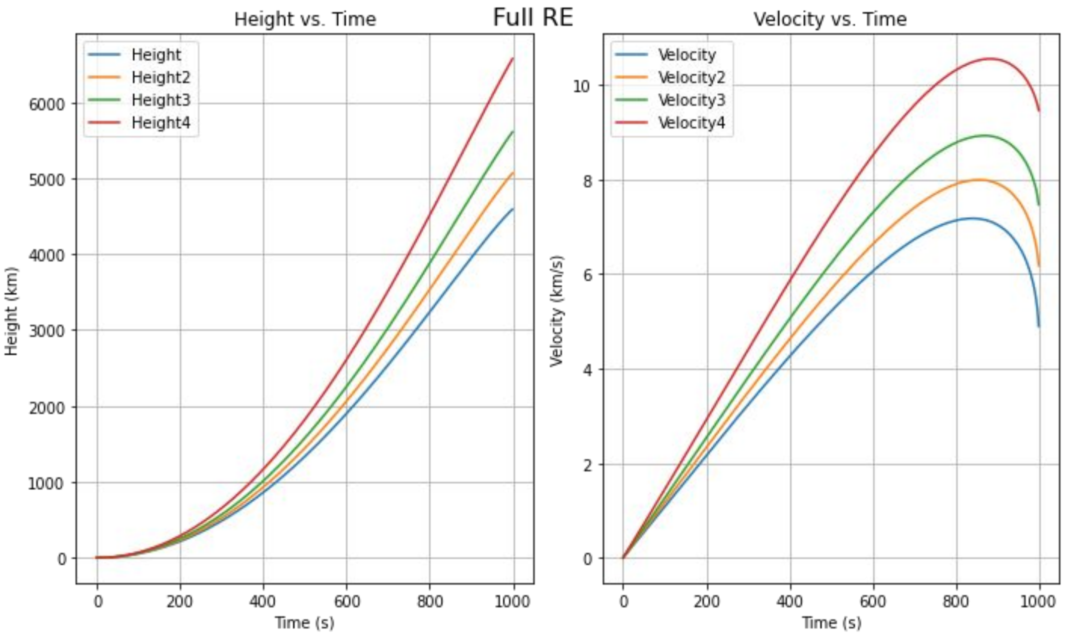
In this case, b=200

The heights and velocities of the rocket are the lowest out of the examples yet

Notice that the velocities of the rocket begin to plateau for the higher exhaust velocities

The rocket reaches terminal velocity and begins to decelerate

RE with Gravity and Friction

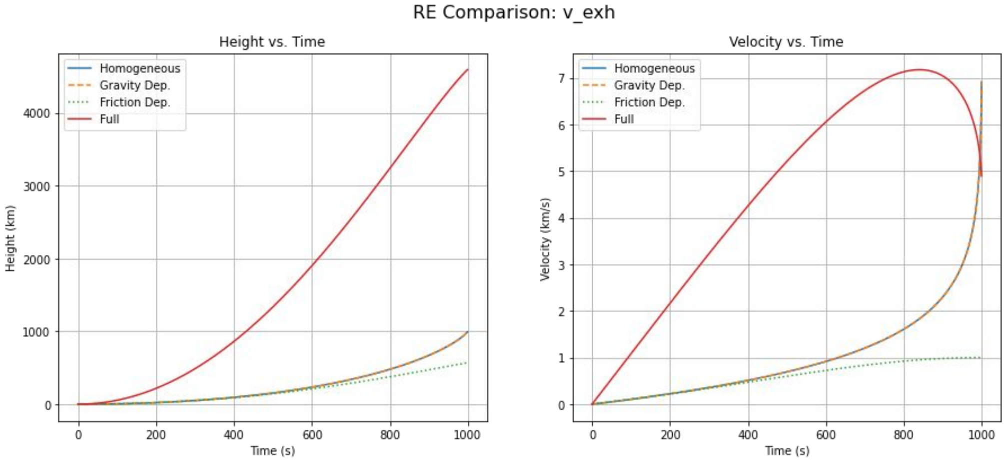


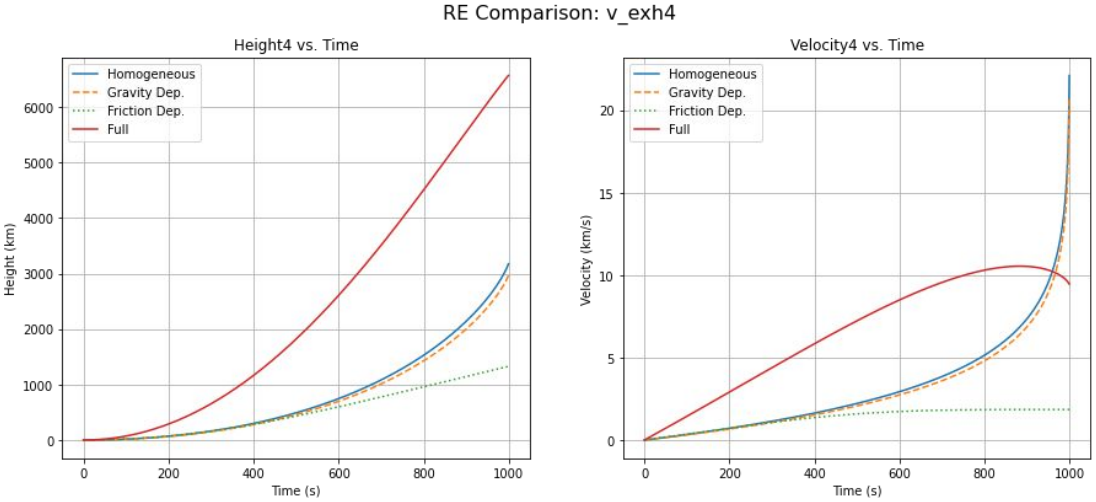
In this case and the rocket is affected by external forces with b=50

Notice that the heights are not extremely low

Although, the velocities begin to decrease after ~800 seconds

Full RE





In these situations, the different lines represent different impacts on the rockets (b=50)

For the graph above, we use v_exh = 1 km/s

For the graph to the left, we use v_exh4 = [2.9, 4,5] km/s

References and Citations

- https://en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation
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