

TOI-561 System

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TOI-561 as a Star

- TOI-561 was discovered RECENTLY in 2020 by Emily Gilbert!
 - Discovered by the transit method i.e. creating observation of a star's light curve dip
- Pretty darn old
 - 10.5 billion years!
- Metal-Poor
 - Implications of a metal-poor star are low metallicity and presence in highly-redshifted areas of the Universe
 - Can we infer anything about its planets? Hm
- Planets
 - TOI-561b, TOI-561c, TOI-561d, TOI-561e, and TOI-561f

Why the TOI-561 System?

- Solar System vs Other Simulations

- As a group we thought going for a solar system would provide a humbling while insightful experience not only for us but for our audience as well!

- Databases available

- We wanted to make sure we could utilize as much information as possible to make the coding more feasible
- The TOI-561 has a lot of known parameters such as
 - Star Mass
 - Star Radius
 - Planet Masses and Radii
 - Orbital Periods and Distances

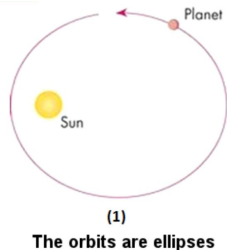
Tackling the Simulation of TOI-561

- What we knew we needed:
 - Values of course!
 - Set of general equations to be used
 - Reference code
- Scaffolding afterwards
 - Utilized past homework (6) and simulation demo to inform our simulation
 - Made use of packages such as: ffmpeg, math, matplotlib
- How to Implement
 - We had reference code, values, and suggestions...but implementing it was a different story

Challenges and Methods Simulating TOI-561

- Methods had to be altered
 - We set off first using object classes - animation lecture demo
 - Ended up using parametric equations for simplicity
- Entailed use of:
 - Physical equations such as Gravitational Force, Kepler's Laws, Eccentricity

$$F = G \frac{m_1 m_2}{r^2}$$



$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

- Mathematical equations in python such as parametric equations using numpy

$$X = a \sin(ct) \quad Y = b \sin(dt) \quad \rightarrow a \text{ and } b = \text{semi-major/minor axis } c \text{ and } d = \text{speed}$$

Methods: Picking System and Initial Calculations

Once we picked which system we were going to use, we went through the database to find out the period in days and the semimajor axis in AU of each planet in the system.

We then found the speeds at which each planet traveled and transformed all the measurements to a more reasonable scale.

For planets with a measurable eccentricity, we calculated the semi-minor axis as well

Handwritten calculations for five planets (1-5) showing orbital parameters:

- Planet 1:**
 - period: 0.44679 days
 - semimajor axis: 0.01055 AU = 1.5
 - velocity: $\approx 2.233 \text{ units/day}$
 - eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.01055^2}{0.44^2}} = 0.9999999999999999$
- Planet 2:**
 - period: 10.779 days
 - semimajor axis: 0.08809 AU = 8.849
 - velocity: $\approx 0.3746 \text{ units/day}$
 - eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.08809^2}{10.779^2}} = 0.8147$
- Planet 3:**
 - period: 16.297 days
 - semimajor axis: 0.1174 AU = 11.12396
 - velocity: $\approx 0.698 \text{ units/day}$
 - eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.1174^2}{16.297^2}} = 0.8510$
- Planet 4:**
 - period: 25.62 days
 - semimajor axis: 0.1569 AU = 15.832
 - velocity: $\approx 0.5805 \text{ units/day}$
 - eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.1569^2}{25.62^2}} = 0.8610$
- Planet 5:**
 - period: 77.23 days
 - semimajor axis: 0.8274 AU = 81.053
 - velocity: $\approx 0.4018 \text{ units/day}$
 - eccentricity: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.8274^2}{77.23^2}} = 0.8610$

Equation for semi-minor axis b :

$$b = \left(\text{semimajor axis} \right) \cos \left(\frac{\text{velocity}}{\text{semimajor axis}} t \right)$$

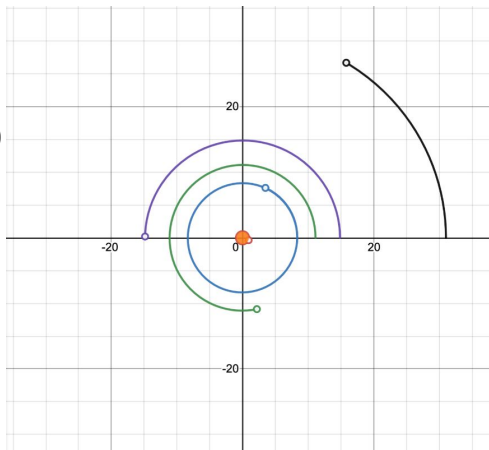
$$y = \left(\text{semimajor axis} \right) \sin \left(\frac{\text{velocity}}{\text{semimajor axis}} t \right)$$

* For eccentricity of 0, semimajor = semiminor

Methods: Final Calculations and Coding

We then transferred all of the information into the parametric equations we described earlier in the background section. To test this out, we used something that was a bit more familiar: desmos

Our final step was to put all of these equations and information into code.



1	$(\cos(2.273t), \sin(2.273t))$	×
	$0 \leq t \leq a$	
2	$(8.349 \cos(\frac{0.7746}{8.349}t), 8.3347 \sin(\frac{0.7746}{8.3347}t))$	×
	$0 \leq t \leq a$	
3	$(11.12796 \cos(\frac{0.683}{11.12796}t), 11.11348 \sin(\frac{0.683}{11.11348}t))$	×
	$0 \leq t \leq a$	
4	$(14.872 \cos(\frac{0.5805}{14.872}t), 14.8443 \sin(\frac{0.5805}{14.8443}t))$	×
	$0 \leq t \leq a$	
5	$(31.033 \cos(\frac{0.4018}{31.033}t), 31.033 \sin(\frac{0.4018}{31.033}t))$	×
	$0 \leq t \leq a$	

End Result...

The Insight Gained from TOI-561

- We found:
 - Toi-561 b orbits the star the fastest → Toi-561 c orbits the star the second fastest → Toi-561 f orbits the star the third fastest → Toi-561 e orbits the star the slowest
 - Inner planets appear to orbit the star the fastest and the planets appear to orbit slower as they move outwards
- Our results:
 - Agrees with Kepler's third law: the larger the semimajor axis, the larger its orbital period

$$p^2 = a^3$$

- For orbits with eccentricity, we used newton's version of Kepler's third law

$$p^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

References and Citations

- We used the following database to make a decision on our planetary system:
 - [NASA Exoplanet Archive](#)
- We used the following resources to improve and inform our code:
 - HW #6 and Animation Demo
 - Paul's Online Notes - [Parametric Equations](#)